LEARNING OUTCOMES (L.O.)

- I can interpret the slope and intercept of a linear model based on context.

Name _____

Lesson 15:
Interpreting Residuals from a Line

Warm Up: Regents Prep

1. Which point is *not* on the graph represented by $y = x^2 + 3x - 6$?

(1) $(-6,12)$ (3) $(2,4)$

(2) $(-4,-2)$ (4) $(3,-6)$

2. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the nth term of this sequence?

(1) $a_n = 8n + 10$ (3) $a_n = 16n + 10$

(2) $a_n = 8n - 14$ (4) $a_n = 16n - 38$

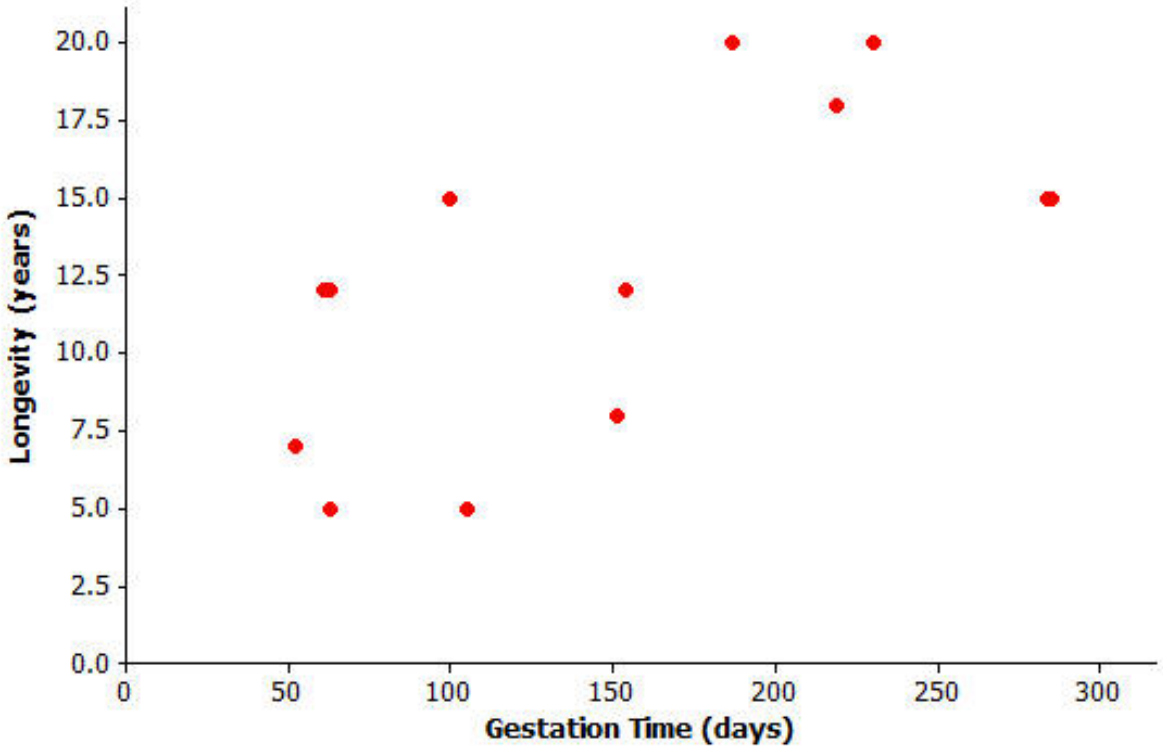
Example 1: Calculating Prediction Errors

The gestation time for an animal is the typical duration between conception and birth. The longevity of an animal is the typical lifespan for that animal. The gestation times (in days) and longevities (in years) for 13 types of animals are shown in the table below.

Animal	Gestation Time (days)	Longevity (years)
Baboon	187	20
Black Bear	219	18
Beaver	105	5
Bison	285	15
Cat	63	12
Chimpanzee	230	20
Cow	284	15
Dog	61	12
Fox (Red)	52	7
Goat	151	8
Lion	100	15
Sheep	154	12
Wolf	63	5

Data Source: *Core Math Tools,* www.nctm.org

Here is the scatter plot for this data set:



1. Calculate the equation of the least-squares line (linear regression) for this data where x represents the gestation time (in days) and y represents longevity in years.

2. Let's specifically investigate how accurately the least-squares line predicted the longevity of the black bear.

   a. What is the gestation time for the black bear?

   b. Look at the graph. Roughly what does the least-squares line predict for the longevity of the black bear?

   c. Use the gestation time from (a) and the least-squares line $y = 6.642 + 0.03974x$ to predict the black bear's longevity. Round your answer to the nearest tenth.

   d. What is the actual longevity of the black bear?

   e. How much do you have to add to the predicted value to get the actual longevity of the black bear?

Example 2: Residuals as Prediction Errors

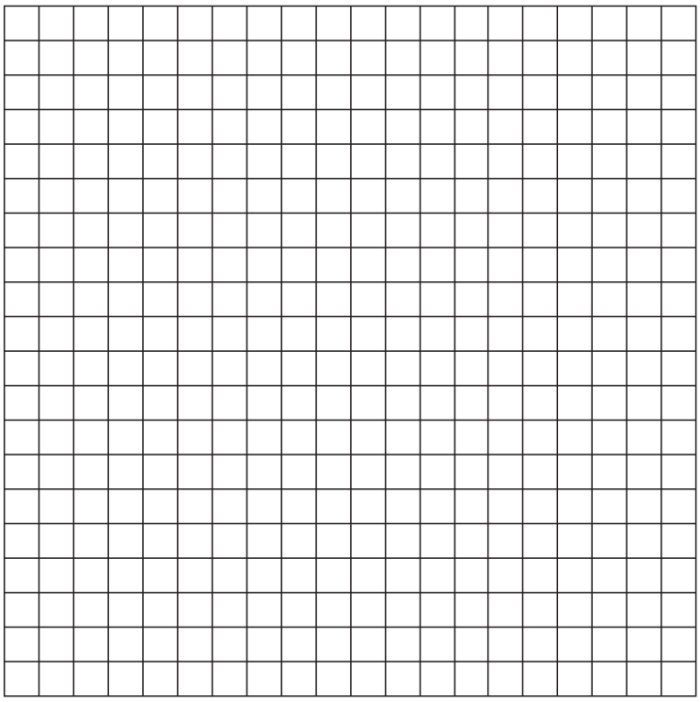
In the example above, you found out how much needs to be added to the predicted value in order to find the true value of the animal's longevity. In order to find this you have calculated the residual.

residual = actual *y* value – predicted *y* value

The values of the residuals are shown in the table below.

Animal	Gestation Time (days)	Longevity (years)	Residual
Baboon	187	20	5.9
Black Bear	219	18	2.7
Beaver	105	5	−5.8
Bison	285	15	−3.0
Cat	63	12	2.9
Chimpanzee	230	20	4.2
Cow	284	15	−2.9
Dog	61	12	2.9
Fox (Red)	52	7	−1.7
Goat	151	8	−4.6
Lion	100	15	4.4
Sheep	154	12	−0.8
Wolf	63	5	−4.1

Create a residual plot of the data.



These residuals show that the actual longevity of an animal should be within six years of the longevity predicted by the least-squares line.

Exercises

Suppose you selected a type of animal that is not included in the original data set, and the gestation time for this type of animal is 270 days. Substituting $x = 270$ into the equation of the least-squares line you get:

$$y = 6.642 + 0.03974(270)$$
$$= 17.4 \text{ years}$$

Think about what the *actual* longevity of this type of animal might be.

1. Could it be 30 years? How about 5 years?

2. Judging by the size of the residuals in our table, what kind of values do you think would be reasonable for the longevity of this type of animal?

3. The gestation time for the type of animal called the ocelot is known to be 85 days.

   The least-squares line predicts the longevity of the ocelot to be:

   $$y = 6.642 + 0.03974(85) = 10.0 \text{ years}$$

   Based on the residuals in Example 2, would you be surprised to find that the longevity of the ocelot was 2 years? Why, or why not? What might be a sensible range of values for the actual longevity of the ocelot?

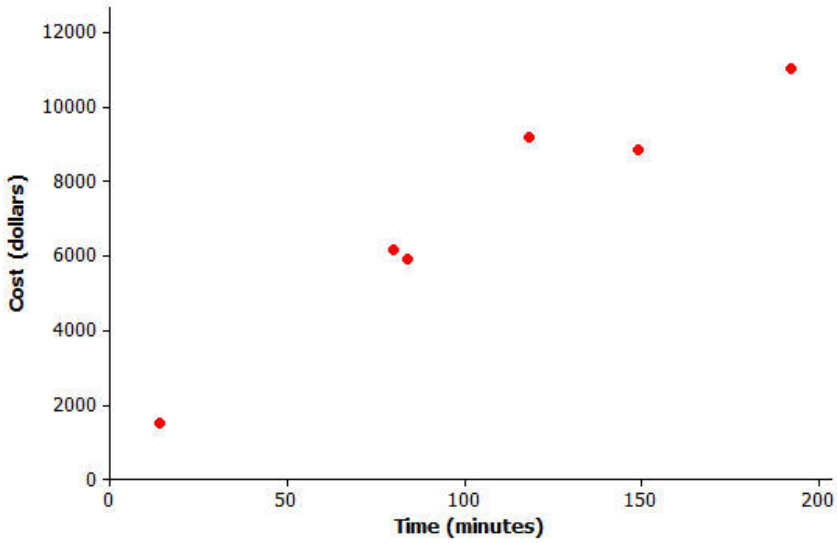
4. We know that the actual longevity of the ocelot is 9 years. What is the residual for the ocelot?

Name _____ Date _____

Lesson 15: Interpreting Residuals from a Line *CW/HW*

The time spent in surgery and the cost of surgery was recorded for six patients. The results and scatter plot are shown below.

Time (minutes)	Cost ($)
14	1,510
80	6,178
84	5,912
118	9,184
149	8,855
192	11,023



1. Calculate the equation of the least-squares line relating cost to time. (*Indicate slope to the nearest tenth and y-intercept to the nearest whole number.*)

2. Draw the least-squares line on the graph above.

3. What does the least-squares line predict for the cost of a surgery that lasts 118 minutes? (Calculate the cost to the nearest cent.)

4. How much do you have to add to your answer to question (3) to get the actual cost of surgery for a surgery lasting 118 minutes? (This is the residual.)

5. Show your answer to question (4) as a vertical line between the point for that person in the scatter plot and the least-squares line.

6. Remember that the residual is the actual y-value minus the predicted y-value. Calculate the residual for the surgery that took 149 minutes and cost $8,855.

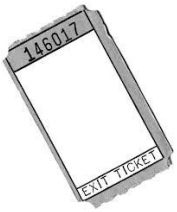
7. Calculate the other residuals, and write all the residuals in the table below.

Time (minutes)	Cost ($)	Predicted value ($)	Residual
14	1,510		
80	6,178		
84	5,912		
118	9,184		
149	8,855		
192	11,023		

8. Suppose that a surgery took 100 minutes.
   a. What does the least-squares line predict for the cost of this surgery?

   b. Would you be surprised if the actual cost of this surgery were $9000? Why or why not?

   c. Interpret the slope of the least-squares line.



Name _____

Lesson 15: Interpreting Residuals from a Line

Meerkats have a gestation time of 70 days.

a. Use the equation of the least-squares line from today's class, or y=6.643+0.03974x, to predict the longevity of the meerkat. Remember x equals the gestation time in days and y equals the longevity in years.

b. Approximately how close might your prediction to be to the actual longevity of the meerkat? What was it (from class) that told you roughly how close a prediction might be to the true value?

c. According to your answers to (a) and (b), what is a reasonable range of possible values for the longevity of the meerkat?

d. The longevity of the meerkat is actually 10 years. Use this value and the predicted value that you calculated in (a) to find the residual for the meerkat.